

二阶脉冲微分方程的反周期解

周邵隆

(广东工业大学 应用数学学院, 广东 广州 510006)

摘要: 在 Hilbert 空间中考虑二阶脉冲方程, 给出其具有反周期解的一个条件.

关键词: 脉冲微分方程; 反周期解; Leray-Schauder 拓扑度

中图分类号: O177.91

文献标识码: A

文章编号: 1007-7162(2009)04-0014-05

1987 年, Hiroko Okochi 在研究非线性抛物方程 $f(t) \in u'(t) + \partial\varphi(u(t))$ 的周期解时^[1-3], 提出其反周期解的存在性问题. 随后许多学者对反周期边值问题进行了深入的研究, 使反周期边值条件出现在各种问题的研究中^[4-7]. 脉冲方程是近几十年发展起来的微分方程的一个重要分支, 它源于生物学和医学的一些数学模型^[8]. 脉冲方程虽然不连续, 但只要定义方程在合适的 Banach 空间以及对其添加适当的条件, 同样具有反周期解.

1 二阶脉冲微分方程的反周期解

考察二阶脉冲方程反周期边值问题:

$$\begin{cases} \mathbf{x}''(t) = \mathbf{A}\mathbf{x}'(t) + f(t, \mathbf{x}), & t \neq t_k (k = 1, 2, \dots, m), \\ \Delta \mathbf{x}|_{t=t_k} = \mathbf{I}_k(\mathbf{x}(t_k)), & (k = 1, 2, \dots, m) \\ \Delta \mathbf{x}'|_{t=t_k} = \bar{\mathbf{I}}_k(\mathbf{x}(t_k), \mathbf{x}'(t_k)), & (k = 1, 2, \dots, m), \\ \mathbf{x}(0) = -\mathbf{x}(1), \quad \mathbf{x}'(0) = -\mathbf{x}'(1) \end{cases} \quad (1)$$

其中 $f \in C[J \times R^n, R^n]$, $J = [0, 1]$, R^n 为 n 维欧氏空间, $C[J \times R^n, R^n]$ 表示映射二维变量 t, x 到 n 维欧氏空间的全体连续函数. \mathbf{A} 为 $n \times n$ 矩阵. $\mathbf{I}_k \in C[R^n, R^n]$, $\bar{\mathbf{I}}_k \in C[R^n \times R^n, R^n]$, $\Delta \mathbf{x}|_{t=t_k} = \mathbf{x}(t_k^+) - \mathbf{x}(t_k^-)$, $\mathbf{x}(t_k^-) = \mathbf{x}(t_k)$, $0 < t_1 < t_2 < \dots < t_m < \dots < t_m < 1$.

令 $PC'[J, R^n] = \{ \mathbf{x}: J \rightarrow R^n \mid \mathbf{x}(t), \mathbf{x}'(t) \text{ 当 } t \neq t_k \text{ 时连续, } \mathbf{x}(t_k^+), \mathbf{x}(t_k^-), \mathbf{x}'(t_k^+), \mathbf{x}'(t_k^-) \text{ 均存在, } k = 1, 2, \dots, m \}$, 易知左导数 $\mathbf{x}'_-(t_k)$ 存在, 且 $\mathbf{x}'_-(t_k) = \mathbf{x}'(t_k^-)$ ^[8].

引入范数 $\| \mathbf{x} \|_{pc} = \max_{t \in J} | \mathbf{x} | + \max_{t \in J} | \mathbf{x}' |$, 则 $PC'[J, R^n]$ 成为一个 Banach 空间. 令 $J_0 = [0, t_1]$, $J_1 = (t_1, t_2], \dots, J_{m-1} = (t_{m-1}, t_m]$, $J_m = (t_m, 1]$, $J' = J \setminus \{t_1, t_2, \dots, t_m\}$. $\mathbf{x} \in PC'[J, R^n] \cap C^2[J', R^n]$ 叫做反

周期边值问题(1)的解, 如果它满足式(1)中的所有等式.

引理 1^[8] $H \subset PC'[J, R^n]$ 是相对紧集的充分必要条件为 H 中诸函数 $x(t)$ 及其导数 $x'(t)$ 都在 J 上一致有界且在每个 $J_k (k = 0, 1, \dots, m)$ 上等度连续.

引理 2^[9] 如果 $\mathbf{x} \in PC'[J, R^n]$, 且满足 $\mathbf{x}(0) = -\mathbf{x}(1)$, $| \mathbf{x}(t_k^+) - \mathbf{x}(t_k^-) | \leq \alpha, k = 1, 2, \dots, m, \alpha > 0$ 为常数, 则 $\| \mathbf{x} \|_{\infty} \leq \frac{1}{2} [\int_0^1 | \mathbf{x}'(t) |^2 dt]^{\frac{1}{2}} + \frac{1}{2} m \alpha$.

证明 当 $t \in [t_1, t_2]$ 时, $\int_0^t \mathbf{x}'(s) ds = \int_{t_1}^t \mathbf{x}'(s) ds +$

$$\int_0^{t_1} \mathbf{x}'(s) ds = \mathbf{x}(t) - (\mathbf{x}(t_1^+) - \mathbf{x}(t_1^-)) - \mathbf{x}(t_1) - \mathbf{x}(0).$$

同理, 当 $t \in [t_k, t_{k+1}]$ 时, $\int_0^t \mathbf{x}'(s) ds = \mathbf{x}(t) - \sum_{0 < t_k < t} (\mathbf{x}(t_k^+) - \mathbf{x}(t_k^-)) - \mathbf{x}(0)$. 又因为 $\mathbf{x}(0) = -\mathbf{x}(1)$ 且 $\int_0^1 \mathbf{x}'(s) ds = \mathbf{x}(1) - \sum_{0 < t_k < 1} (\mathbf{x}(t_k^+) - \mathbf{x}(t_k^-)) - \mathbf{x}(0)$, 所以 $\mathbf{x}(0) = -\frac{1}{2} \int_0^1 \mathbf{x}'(s) ds - \frac{1}{2} \sum_{0 < t_k < 1} (\mathbf{x}(t_k^+) - \mathbf{x}(t_k^-))$.

$$\begin{aligned} \mathbf{x}(t) &= \int_0^t \mathbf{x}'(s) ds + \sum_{0 < t_k < t} (\mathbf{x}(t_k^+) - \mathbf{x}(t_k^-)) - \\ &\frac{1}{2} \int_0^1 \mathbf{x}'(s) ds - \frac{1}{2} \sum_{0 < t_k < t} (\mathbf{x}(t_k^+) - \mathbf{x}(t_k^-)) = \\ &\frac{1}{2} \int_0^t \mathbf{x}'(s) ds - \frac{1}{2} \int_0^1 \mathbf{x}'(s) ds + \frac{1}{2} \sum_{0 < t_k < t} (\mathbf{x}(t_k^+) - \\ &\mathbf{x}(t_k^-)) - \frac{1}{2} \sum_{0 < t_k < t} (\mathbf{x}(t_k^+) - \mathbf{x}(t_k^-)). \end{aligned}$$

$$\begin{aligned} \| \mathbf{x} \|_{\infty} &\leq \frac{1}{2} \int_0^1 | \mathbf{x}'(s) | ds + \frac{1}{2} \int_0^1 | \mathbf{x}'(s) | ds + \\ &\frac{1}{2} \sum_{0 < t_k < t} | (\mathbf{x}(t_k^+) - \mathbf{x}(t_k^-)) | + \frac{1}{2} \sum_{0 < t_k < t} | (\mathbf{x}(t_k^+) - \end{aligned}$$

收稿日期: 2009-06-03

作者简介: 周邵隆(1982-), 男, 硕士研究生, 主要研究方向为非线性泛函分析.

$$x(t_k)) \leq \frac{1}{2} \int_0^1 |x'(s)| ds + \frac{1}{2} \sum_{0 < t_k < t} (x(t_k^+) - x(t_k))$$

$$x(t_k)) \leq \frac{1}{2} \left[\int_0^1 |x'(s)|^2 ds \right]^{\frac{1}{2}} + \frac{1}{2} m\alpha.$$

引理3 如果 $x \in PC'[J, R^n]$, 且满足方程(1), $|f(t, x)| \leq a|x(t)| + |g(t)|, a > 0, \|g\|_\infty < \infty, |x'(t_k^+) - x'(t_k)| \leq \beta, k = 1, 2, \dots, m, \beta > 0$ 为常数, 则

$$\|x'\|_\infty \leq \left(\|A\| + \frac{a}{2} \right) \left(\int_0^1 |x'(s)|^2 ds \right)^{\frac{1}{2}} + \|g\|_\infty + m\beta + m\alpha.$$

证明 设 $x \in PC'[J, R^n]$ 满足 $x''(t) = Ax'(t) + f(t, x), t \neq t_k (k = 1, 2, \dots, m)$, 则

$$x'(t) = -\frac{1}{2} \int_0^1 Ax'(s) + f(s, x) ds - \frac{1}{2} \sum_{0 < t_k < t} (x'(t_k^+) - x'(t_k)) + \int_0^t Ax'(s) + f(s, x) ds + \sum_{0 < t_k < t} (x'(t_k^+) - x'(t_k)),$$

$$x'(t) = \frac{1}{2} \int_0^1 Ax'(s) + f(s, x) ds - \frac{1}{2} \int_t^1 Ax'(s) + f(s, x) ds + \frac{1}{2} \sum_{0 < t_k < t} (x'(t_k^+) - x'(t_k)) + \frac{1}{2} \sum_{0 < t_k < t} (x'(t_k^+) - x'(t_k)),$$

于是 $\|x'(t)\|_\infty \leq \int_0^1 |Ax'(s) + f(s, x)| ds + \sum_{0 < t_k < t} |x'(t_k^+) - x'(t_k)| \leq \|A\| \int_0^1 |x'(s)| ds + \int_0^1 |f(s, x)| ds + \sum_{0 < t_k < t} |x'(t_k^+) - x'(t_k)| \leq \|A\| \left(\int_0^1 |x'(s)|^2 ds \right)^{\frac{1}{2}} + a\|x\|_\infty + \|g\|_\infty + m\beta.$

由引理2可知

$$\|x'\|_\infty \leq \left(\|A\| + \frac{a}{2} \right) \left(\int_0^1 |x'|^2 ds \right)^{\frac{1}{2}} + \|g\|_\infty + m\beta + m\alpha.$$

定理1 设 $A: R^n \rightarrow R^n$ 为 $n \times n$ 矩阵, $f(t+1, x) = -f(t, x)$ 且 f 满足如下条件:

- 1) $|f(t, x)| \leq a|x(t)| + |g(t)|, a > 0, \|g\|_\infty < \infty.$
- 2) $|\Delta x|_{t=t_k} = |x(t_k^+) - x(t_k)| \leq \alpha, |\Delta x'|_{t=t_k} = |x'(t_k^+) - x'(t_k)| \leq \beta.$
- 3) $a + 2\|A\| < 4.$

则方程(1)至少有一解.

条件2)事实上只需考虑函数 I_k 及 \bar{I}_k 是否一致有界, 若 I_k, \bar{I}_k 一致有界, 则显然满足 $|I_k(x(t_k))| = |\Delta x(t_k)|_{t=t_k} = |x(t_k^+) - x(t_k)| \leq \alpha, |\bar{I}_k(x(t_k))|$

$$x'(t_k))| = |\Delta x'(t_k)|_{t=t_k} = |x'(t_k^+) - x'(t_k)| \leq \beta, \alpha, \beta > 0.$$

证明 设 x 满足 $x''(t) = Ax'(t) + f(t, x), t \neq t_k, (k = 1, 2, \dots, m)$, 则证明方程(1)有解等价于证明方程

$$x(t) = x(0) + x'(0)t - \int_0^t (t-s)[Ax'(s) + f(s, x) ds + \sum_{0 < t_k < t} (x(t_k^+) - x(t_k)) + \sum_{0 < t_k < t} (x'(t_k^+) - x'(t_k))(t-t_k), t \in J'] \quad (2)$$

有解. 其中

$$x(0) = -\frac{1}{2} \int_0^1 x'(s) ds - \frac{1}{2} \sum_{0 < t_k < 1} (x(t_k^+) - x(t_k)),$$

$$x'(0) = -\frac{1}{2} \int_0^1 Ax'(s) + f(s, x) ds - \frac{1}{2} \sum_{0 < t_k < 1} (x'(t_k^+) - x'(t_k)).$$

定义算子 $K: PC'[J, R^n] \rightarrow PC'[J, R^n]$ 如下:

$$Kx(t) = x(0) + x'(0)t - \int_0^t (t-s)[Ax'(s) + f(s, x)] ds + \sum_{0 < t_k < t} (x(t_k^+) - x(t_k)) + \sum_{0 < t_k < t} (x'(t_k^+) - x'(t_k))(t-t_k), t \in J',$$

则算子 K 是映 $PC'[J, R^n]$ 入 $PC'[J, R^n]$ 的全连续算子.

先证 K 连续. 设 $\|x_n(t) - x(t)\|_{PC'} \rightarrow 0$, 又有 I_k, \bar{I}_k, f 连续, 所以

$$\|x_n(0) - x(0)\|_\infty \leq \frac{1}{2} \int_0^1 \|x'_n(s) - x'(s)\|_\infty ds + \frac{1}{2} \sum_{k=1}^m |I_k(x_n(t_k) - I_k x(t_k))| \leq \frac{1}{2} \|x'_n(s) - x'(s)\|_{PC'} + \frac{1}{2} m\epsilon \rightarrow 0, \|x'_n(0) - x'(0)\|_\infty \leq \frac{1}{2} \int_0^1 \|Ax'_n(s) - Ax'(s)\|_\infty + \|f(s, x_n) - f(s, x)\|_\infty ds + \frac{1}{2} \sum_{k=1}^m |\bar{I}_k(x'_n(t), x_n(t) - \bar{I}_k(x'(t), x(t))| \leq \frac{1}{2} \|Ax'(s) - Ax'(s)\|_{PC'} + \frac{1}{2} \|f(s, x_n) - f(s, x)\|_\infty + \frac{1}{2} m\epsilon \rightarrow 0.$$

所以

$$\|Kx_n - Kx\|_\infty \leq \|(x_0)_n(t) - x_0(t)\|_\infty + \|(x'_0)_n(t) - x'_0(t)\|_\infty + \int_0^1 \|Ax'_n(s) - Ax'(s)\|_\infty + \|f(s, x_n) - f(s, x)\|_\infty ds + \sum_{k=1}^m |I_k(x_n(t_k) - I_k(x(t_k)))| + \sum_{k=1}^m |\bar{I}_k(x'_n(t), x_n(t) - \bar{I}_k(x'(t), x(t))| \leq 2\epsilon +$$

$$\begin{aligned} & \|Ax'_n(s) - Ax'(s)\|_{PC'} + \\ & \|f(s, x_n) - f(s, x_n)\|_{\infty} + 2m\varepsilon \rightarrow 0. \end{aligned}$$

于是 K 连续.

下证 K 是紧算子. 设 X 是 $PC'[J, R^n]$ 中的有界集, 则 $X = \{x \mid \|x\|_{PC'} < \infty, x \in PC'[J, R^n]\}$.

显然 $\|x(0) + x'(0)t\|_{\infty} \leq \|x\|_{\infty} + \|x'\|_{\infty}t \leq \|x\|_{PC'}$, 所以 $\|Kx(t)\|_{PC'} \leq \|x(t)\|_{PC'} + \|A\| \|x(t)\|_{PC'} + a \|x(t)\|_{PC'} + \|g\|_{\infty} + \sum_{0 < t_k < 1} |(x'(t_k^+) - x'(t_k))| +$

$$\sum_{0 < t_k < 1} |(x'(t_k^+) - x'(t_k))| < \infty,$$

所以 $Kx(t)$ 和 $Kx'(t)$ 在 J 上一致有界.

又因为

$$\begin{aligned} \|Kx''(t)\|_{\infty} &= \|Ax'(t) + f(t, x)\|_{\infty} \leq \\ &\|A\| \|x\|_{PC'} + a \|x\|_{PC'} + \|g\|_{\infty} \leq \infty, \end{aligned}$$

所以 K 在每个 $J_k (k=0, 1, \dots, m)$ 上等度连续. 由引理 1 可知 K 映有界集到相对紧集, 又 K 连续, 所以 K 是全连续算子^[8-9].

因此, 为证明二阶脉冲方程(1)有解, 只需证明存在常数 $M > 0$, 使得当 $\lambda \in (0, 1)$, x 是二阶脉冲方程 $x''(t) = \lambda(Ax'(t) + f(t, x)), t \neq t_k (k=1, 2, \dots, m)$ 的解时, 必有 $\|x\|_{PC'} \leq M^{[9-10]}$.

设 x 是方程 $x''(t) = \lambda(Ax'(t) + f(t, x)), t \neq t_k (k=1, 2, \dots, m)$ 的解.

$$\begin{aligned} & \text{用 } x(t) \text{ 对上式方程两边作内积, 得 } \int_0^1 x''(t) dt = \\ & \lambda \int_0^1 Ax'(t)x(t) + f(t, x)x(t) dt. \end{aligned}$$

由分部积分知

$$\begin{aligned} \int_0^1 x''(t)x(t) &= x'(1)x(1) - \sum_{0 < t_k < 1} (x'(t_k^+)x(t_k^+) - \\ & x'(t_k)x(t_k)) - x'(0)x(0) - \int_0^1 (x'(t))^2 dt. \end{aligned}$$

注意到 $x'(1)x(1) = x'(0)x(0)$, 可得

$$\begin{aligned} \int_0^1 (x'(t))^2 dt &= \sum_{0 < t_k < 1} (x'(t_k^+)x(t_k^+) - x'(t_k)x(t_k)) - \\ & \lambda \int_0^1 Ax'(t)x(t) + f(t, x)x(t) dt, \end{aligned}$$

$$\int_0^1 (x'(t))^2 dt \leq \left| \sum_{0 < t_k < 1} (x'(t_k^+)x(t_k^+) - x'(t_k)x(t_k)) \right| +$$

$$\lambda \int_0^1 |Ax'(t)x(t) + f(t, x)x(t)| dt \leq$$

$$\left| \sum_{0 < t_k < 1} (x'(t_k^+)x(t_k^+) - x'(t_k)x(t_k)) \right| +$$

$$\int_0^1 \|Ax'(t)x(t)\| dt + \int_0^1 |f(t, x)x(t)| dt \leq$$

$$\left| \sum_{0 < t_k < 1} (x'(t_k^+)x(t_k^+) - x'(t_k)x(t_k)) \right| +$$

$$\|A\| \|x(t)\|_{\infty} \int_0^1 |x'(t)| dt + \int_0^1 a |x'(t)|^2 +$$

$$\begin{aligned} & |g(t)x(t)| dt \leq \left| \sum_{0 < t_k < 1} (x'(t_k^+)x(t_k^+) - \right. \\ & \left. x'(t_k)x(t_k)) \right| + \|A\| \|x(t)\|_{\infty} \left(\int_0^1 |x'(t)|^2 dt \right)^{\frac{1}{2}} + \\ & a \|x(t)\|_{\infty}^2 + \|g\|_{\infty} \|x(t)\|_{\infty}. \end{aligned}$$

由引理 2 可知

$$\begin{aligned} \int_0^1 (x'(t))^2 dt &\leq \left| \sum_{0 < t_k < 1} (x'(t_k^+)x(t_k^+) - x'(t_k)x(t_k)) \right| + \\ &\|A\| \left(\frac{1}{2} \left(\int_0^1 |x'(s)|^2 ds \right)^{\frac{1}{2}} + \right. \\ &\left. \frac{1}{2} m\alpha \right) \left(\int_0^1 |x'(t)|^2 dt \right)^{\frac{1}{2}} + \\ &a \left(\frac{1}{2} \left(\int_0^1 |x'(s)|^2 ds \right)^{\frac{1}{2}} + \frac{1}{2} m\alpha \right) + \\ &\|g\|_{\infty} \left(\frac{1}{2} \left(\int_0^1 |x'(s)|^2 ds \right)^{\frac{1}{2}} + \frac{1}{2} m\alpha \right). \end{aligned}$$

由于

$$\begin{aligned} & \left| \sum_{0 < t_k < 1} (x'(t_k^+)x(t_k^+) - x'(t_k)x(t_k)) \right| \leq \\ & \sum_{0 < t_k < 1} |x'(t_k^+)x(t_k^+) - x'(t_k)x(t_k^+) + x'(t_k)x(t_k^+) - \\ & x'(t_k)x(t_k)| \leq \sum_{0 < t_k < 1} |x'(t_k^+) - x'(t_k)| |x'(t_k^+)| + \\ & |x'(t_k^+) - x'(t_k)| |x'(t_k)|, \end{aligned}$$

$$\begin{aligned} & \text{而 } |x'(t_k^+)| \leq |x'(t_k)| + |x'(t_k^+) - x'(t_k)| \leq \\ & \|x'\|_{\infty} + \alpha \leq \frac{1}{2} \left[\int_0^1 |x'(s)|^2 ds \right]^{\frac{1}{2}} + \frac{1}{2} m\alpha, (k=1, \\ & 2, \dots, m), \end{aligned}$$

$$\begin{aligned} & \text{又 } |x'(t_k)| \leq \|x'\|_{\infty} \leq \left(\|A\| + \frac{a}{2} \right) \left(\int_0^1 |x'(s)|^2 ds \right)^{\frac{1}{2}} + \\ & \|g\|_{\infty} + m\beta + m\alpha, (k=1, 2, \dots, m), \end{aligned}$$

所以

$$\begin{aligned} & \left| \sum_{0 < t_k < 1} (x'(t_k^+)x(t_k^+) - x'(t_k)x(t_k)) \right| \leq \\ & m\beta \left(\frac{1}{2} \left[\int_0^1 |x'(s)|^2 ds \right]^{\frac{1}{2}} + \frac{1}{2} m\alpha + \alpha \right) + \\ & m\alpha \left(\left(\|A\| + \frac{a}{2} \right) \left(\int_0^1 |x'(s)|^2 ds \right)^{\frac{1}{2}} + \right. \\ & \left. \|g\|_{\infty} + m\beta + m\alpha \right), \end{aligned}$$

整理得

$$\begin{aligned} & \left(1 - \frac{1}{2} \|A\| - \frac{1}{4} a \right) \int_0^1 (x'(t))^2 dt - \\ & \left(\frac{3}{2} \|A\| m\alpha + a m\alpha + \frac{1}{2} \|g(t)\|_{\infty} + \right. \\ & \left. \frac{1}{2} m\beta \right) \left(\int_0^1 (x'(t))^2 dt \right)^{\frac{1}{2}} - \left(m\alpha \left(\frac{1}{2} + \right. \right. \\ & \left. \left. \|g\|_{\infty} \right) + \frac{5}{4} m^2 \alpha^2 + \frac{3}{2} m^2 \alpha \beta + m\alpha \beta \right) \leq 0. \end{aligned}$$

$$\text{因为 } - \left(m\alpha \left(\frac{1}{2} + \|g\|_{\infty} \right) + \frac{5}{4} m^2 \alpha^2 + \frac{3}{2} m^2 \alpha \beta + \right.$$

$m\alpha\beta) \leq 0$, 且 $1 - \frac{1}{2} \|A\| - \frac{1}{4}a > 0$, 所以由抛物线的性质可知, 必存在 $M > 0$ 使得 $(\int_0^1 (x'(t))^2 dt)^{\frac{1}{2}} < M$. 再由引理 2 可知

$$\|x\|_{\infty} \leq \frac{1}{2}M + \frac{1}{2}m\alpha.$$

$$\text{又 } x'(t) = x'(0) + \lambda \int_0^t Ax'(s) + f(s, x) ds + \sum_{0 < t_k < t} (x'(t_k^+) - x'(t_k)),$$

$$\text{注意到 } |x'(0)| \leq \frac{1}{2} \int_0^1 |Ax'(s) + f(s, x)| ds +$$

$$\frac{1}{2} \sum_{0 < t_k < 1} |x'(t_k^+) - x'(t_k)|,$$

所以

$$\|x'(t)\|_{\infty} \leq |x'(0)| + \int_0^1 |Ax'(s) + f(s, x)| ds +$$

$$\sum_{0 < t_k < 1} |x'(t_k^+) - x'(t_k)| \leq \frac{3}{2} \int_0^1 |Ax'(s) +$$

$$f(s, x)| ds + \frac{3}{2} \sum_{0 < t_k < 1} |x'(t_k^+) - x'(t_k)| \leq$$

$$\frac{3}{2} (\|A\| (\int_0^1 (x'(s))^2 ds)^{\frac{1}{2}} + a \|x(t)\|_{\infty} +$$

$$\|g(t)\|_{\infty}) + \frac{3}{2}m\beta \leq \frac{3}{2} (\|A\| (M +$$

$$a(\frac{1}{2}M + \frac{1}{2}m\alpha) + \|g(t)\|_{\infty}) + \frac{3}{2}m\beta.$$

$$\text{令 } M_1 = \frac{3}{2} (\|A\| (M + a(\frac{1}{2}M + \frac{1}{2}m\alpha) + \|g(t)\|_{\infty}) +$$

$$\frac{3}{2}m\beta + \frac{1}{2}M + \frac{1}{2}m\alpha, \text{ 则 } \|x\|_{PC} \leq M_1. \text{ 根据 Leray-}$$

Schauder 原理, 可知方程(1)有解. 证完.

例 考虑下列反周期边值问题

$$\begin{cases} x''_1 = \frac{1}{4}x'_1 + \frac{1}{2}x'_2 + \frac{1}{1+x_2^2} + x_1 \cos 2\pi t, & t \in [0, 1], \\ x''_2 = \frac{1}{2}x'_1 - \frac{1}{3}x'_2 + \frac{1}{1+x_1^2} + x_2 \sin 2\pi t, & t \in [0, 1], \\ \Delta x'_1(1/2) = \frac{1}{1 + |x'_1(1/2)| \|x_2(1/2)\|}, \\ \Delta x'_2(1/2) = \frac{1}{6 + 5|x_1(1/2)|}, \\ \Delta x_1(1/2) = \frac{1}{1 + (x_2(1/2))^2}, \\ \Delta x_2(1/2) = \frac{1}{1 + |x_1(1/2)| \|x_2(1/2)\|}, \\ x_1(0) = -x_1(1), \\ x_2(0) = -x_2(1). \end{cases}$$

$$\text{设 } x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, f(t, x) = \begin{pmatrix} \frac{1}{1+x_2^2} + x_1 \cos 2\pi t \\ \frac{1}{1+x_1^2} + x_2 \sin 2\pi t \end{pmatrix},$$

$$A = \begin{pmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{3} \end{pmatrix},$$

$$I(x) = \begin{pmatrix} \frac{1}{1 + (x_2(1/2))^2} \\ \frac{1}{1 + |x_1(1/2)| \|x_2(1/2)\|} \end{pmatrix},$$

$$\bar{I}(x) = \begin{pmatrix} \frac{1}{1 + |x'_1(1/2)| \|x_2(1/2)\|} \\ \frac{1}{6 + 5|x_1(1/2)|} \end{pmatrix}$$

易知 $\|A\| \leq \frac{7}{6}, |\bar{I}(x)| \leq 1, |f(t, x)| \leq |x| + 1$.

由定理 1 知上述方程有解.

参考文献:

- [1] Okochi H. On the existence of periodic solutions to nonlinear abstract parabolic equations [J]. J Math Soc, 1988, (40): 541-553.
- [2] Okochi H. On the existence of anti-periodic solutions to a nonlinear evolution equation associated with differential operators [J]. J Funct Anal, 1990, (91): 246-258.
- [3] Okochi H. On the existence of anti-periodic solutions to nonlinear parabolic equations in noncylindrical domains [J]. Nonlinear Anal, 1990(14): 771-783.
- [4] Haraux A. Anti-periodic solutions of some nonlinear evolution equations [J]. Manuscripta math, 1989(63): 479-505.
- [5] Chen Y Q. Note on Massera's theorem on anti-periodic solution [J]. Advances in Math Sci and Appl, 1999(9): 125-128.
- [6] Chen Y Q, Cho Y J, O'Regan D. Antiperiodic solutions for evolution equations with mapping in class (S+) [J]. Math Nachr, 2005, 278: 335-362.
- [7] Chen Y Q. Antiperiodic solutions for semilinear evolution equations [J]. J Math Anal Appl, 2006, 315: 337-348.
- [8] 郭大均, 孙经先, 刘兆理. 非线性常微分方程泛函方法 [M]. 山东: 山东科学技术出版社, 2006.
- [9] Chen Y Q, Wang X D, Xu H X. Anti-periodic solutions for semilinear evolution equations [J]. J Math Anal Appl, 2002, 273: 627-636.
- [10] Chen Y Q, Cho Y J, Jung J S. Antiperiodic solutions for semilinear evolution equations [J]. Mathematical and Computer Modeling, 2004, 40: 1123-1130.

(下转第 21 页)

参考文献:

- [1] Bondy J A, Murty U S R. Graph Theory with Applications [M]. London: London Macmillan Press, 1976.
- [2] Lovasz L, Plummer M D. Matching Theory [M]. Holland: B V North Holland Elsevier Science Publishers, 1985.
- [3] Wang Chunxiang, Fei Pusheng. Independence number in claw-free graphs [J]. Acta Mathematica Scientia, 2009, 29 (1): 114-121.
- [4] Allan R B, Laskar R C. On domination and independent domination numbers of a graph [J]. Discrete Mathematics, 1978, 23: 73-76.

Number of Perfect Matchings of Claw-free Cubic Graphs without $K_4 - e$ as Its Induced Subgraphs

Yang Chun-xia

(Huali College, Guangdong University of Technology, Guangzhou 511325, China)

Abstract: Let G be a simple graph with vertex set $V(G)$ and edge set $E(G)$. A graph G is said to be claw-free if G does not contain any induced subgraph isomorphic to $K_{1,3}$. A cubic graph is a graph, each vertex of which has degree three. The perfect matching number of claw-free cubic graphs without $K_4 - e$ as its induced subgraphs is characterized.

Key words: perfect matching; claw-free graphs; cubic graphs

(上接第 17 页)

On Anti-periodic Solutions to Second Order Impulsive Differential Equations

Zhou Shao-long

(Faculty of Applied Mathematics, Guangdong University of Technology, Guangzhou 510006, China)

Abstract: It discusses the existence problems with anti-periodic solutions to second order impulsive differential equations in Hilbert space.

Key words: pulsed differential equation; anti-periodic solutions; Leray-Schauder topological degree